This method may also be used where the equations are symmetric with respect to x and $-y$. In this case we have

$$x-y=-p,\ xy=-q,\ \text{and}\ x^2+y^2=p^2-2q.$$

By calculating other symmetric functions the method can be used successfully in solving many equations of higher degree than the second.

SOME CONSTRUCTIONS LEADING TO CONICS.

By F. H. HODGE, Franklin College, Indiana.

Among the courses that find a place in collegiate mathematics one is usually given which involves the treatment of plane curves. Certain loci are described in such a course and their equations are derived and the properties indicated by the equations are discussed. It seems desirable to have a stock of plane loci which can be simply described, and the derivation of whose equations does not present great difficulty, but which are not treated in current text books. Such loci could be assigned to pupils as original exercises, and they would serve to stimulate the capacity for independent thought.

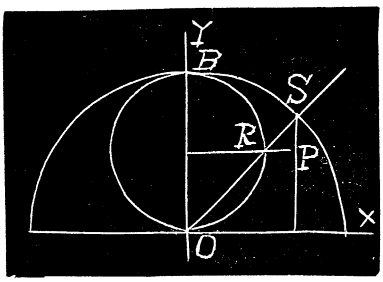

Fig. 1.

The four constructions which follow lead to conics. These are so simple and so directly analogous to a well known construction for the ellipse that it is scarcely possible that they are new though they are not found in current texts. The first two are described in detail, and the others merely suggested.

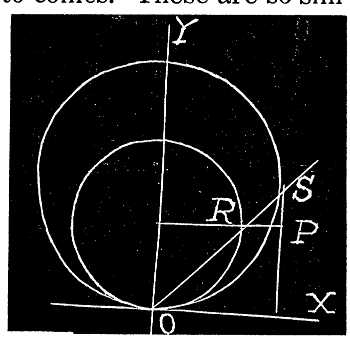

Fig. 2.

(1) Given two circles tangent internally at B and having the radius of the larger equal to the diameter of the smaller. Take the center of the larger circle as origin and the tangent to the smaller circle at that point as the x-axis, the y-axis being perpendicular to the x-axis. Draw a secant line through the origin, meeting the smaller circle at R and the larger circle at S. Through R draw a line parallel to the x-axis, and through S draw a line parallel to the y-axis. Call the point P in which these two lines meet. Required to find the locus of P as the secant line revolves about the origin as an axis. See Fig. 1.

(2) Here again are given two circles tangent internally, but no definite ratio is specified between the lengths of the radii. Take the point of tangency as the origin and the common tangent to the two circles as the x-axis, the y-axis being perpendicular to the x-axis. Draw the secant line as before and also the lines through R and S parallel to the x- and y-axes, respectively, and meeting at P. Required the locus of P as the secant line revolves. See Fig. 2.

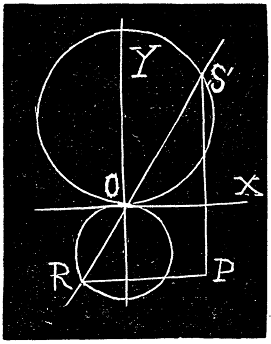

Fig. 3.

(3) This construction is similar to that of (2) except that the two circles are tangent externally, and the common tangent to the circles at the point of tangency is taken as the x-axis. See Fig. 3.

(4) Here the center of one circle lies on the circumference of the other circle. This center is taken as the origin, and the line joining it to the center of the other circle is taken as the y-axis. The secant and the other lines are drawn as in the other constructions, and the locus of P is required. See Fig. 4.

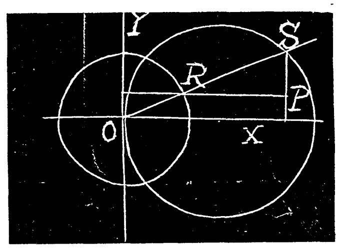

Fig. 4.

All four of these constructions can be shown by very elementary considerations to lead to conics, two of them giving ellipses, and the other two giving parabolas. Constructions similar to these can be made to yield an indefinite number of curves by simply replacing one or both of the circles by ellipses, parabolas, hyperbolas, or other curves. Such considerations, however, would lead us beyond the scope of an elementary course.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

368. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the functional equation, $\dfrac{f(-x)}{f(x)}=r^{2x}$.

I. Solution by the PROPOSER.

Let $f(x)=A(x^2)+xB(x^2)$. Then the equation becomes